

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

1. Let  $f : X \rightarrow Y$ . Let  $\sim$  be a relation on  $X$  by  $x \sim y$  if and only if  $f(x) = f(y)$ . Show  $\sim$  is an equivalence relation on  $X$ .

2. Let  $\mathcal{F}$  be a family of sets and let  $\preceq$  be a relation on  $\mathcal{F}$  by  $X \preceq Y$  if and only if  $X \subseteq Y$ . Show  $\preceq$  is a partial order on  $\mathcal{F}$ .

3. Prove that  $\sqrt{7}$  is irrational.

4. Prove there exists irrational numbers  $x$  and  $y$  such that  $x^y$  is rational.

5. Prove that for any  $n \in \mathbb{N}$  the following holds:

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k$$

6. Let  $n \in \mathbb{N}$ . Prove that

$$\sum_{k=0}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

7. Prove that for any  $n \in \mathbb{N}$ ,  $2^n > n$  holds.

8. Let  $P$  and  $Q$  be statements. Prove that  $(P \implies Q) \iff (\neg P \vee Q)$ .